# Introduction to Complex Numbers

## Classwork

### Opening Exercise

Solve each equation for $x$.

a. $x - 1 = 0$

b. $x + 1 = 0$

c. $x^2 - 1 = 0$

d. $x^2 + 1 = 0$

### Example 1: Addition with Complex Numbers

Compute $(3 + 4i) + (7 - 20i)$.

### Example 2: Subtraction with Complex Numbers

Compute $(3 + 4i) - (7 - 20i)$.

**Lesson Summary**

Every complex number is in the form $a + bi$, where $a$ is the real part and $b$ is the imaginary part of the number. Real numbers are also complex numbers; the real number $a$ can be written as the complex number $a + 0i$. Numbers of the form $bi$, for real numbers $b$, are called imaginary numbers.

Adding two complex numbers is analogous to combining like terms in a polynomial expression.

Multiplying two complex numbers is like multiplying two binomials, except one can use $i^2 = -1$ to further write the expression in simpler form.

Complex numbers satisfy the associative, commutative, and distributive properties.

Complex numbers allow us to find solutions to polynomial equations that have no real number solutions.

## Example 3: Multiplication with Complex Numbers

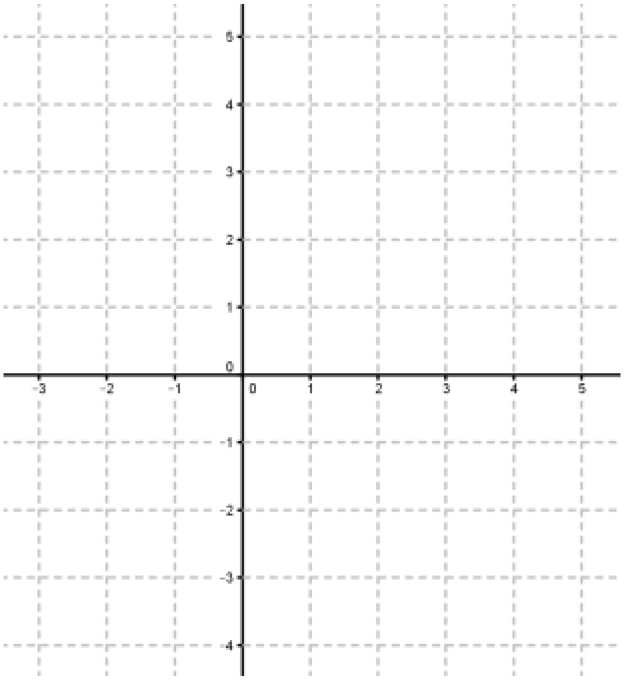
Compute $(1 + 2i)(1 - 2i)$.

## Example 4: Multiplication with Complex Numbers

Verify that $-1 + 2i$ and $-1 - 2i$ are solutions to $x^2 + 2x + 5 = 0$.

## Practice Work:

1. Plot the point on the complex plane corresponding to the complex number given in parts (a)–(h). On one set of axes, label each point by its identifying letter. For example, the point corresponding to $5+2i$ should be labeled $a$.
   a. $5+2i$
   b. $3-2i$
   c. $-2-4i$
   d. $-i$
   e. $\frac{1}{2}+i$
   f. $\sqrt{2}-3i$
   g. $0$
   h. $-\frac{3}{2}+\frac{\sqrt{3}}{2}i$



2. Express each of the following in $a+bi$ form.
   a. $(13+4i)+(7+5i)$

   b. $(5-i)-2(1-3i)$

   c. $\left((5-i)-2(1-3i)\right)^2$

   d. $(3-i)(4+7i)$

   e. $(3-i)(4+7i)-\left((5-i)-2(1-3i)\right)$

3. Express each of the following in $a + bi$ form.
   a. $(2 + 5i) + (4 + 3i)$

   b. $(-1 + 2i) - (4 - 3i)$

   c. $(4 + i) + (2 - i) - (1 - i)$

   d. $(5 + 3i)(3 + 5i)$

   e. $-i(2 - i)(5 + 6i)$

   f. $(1 + i)(2 - 3i) + 3i(1 - i) - i$

4. Find the real values of $x$ and $y$ in each of the following equations using the fact that if $a + bi = c + di$, then $a = c$ and $b = d$.
   a. $5x + 3yi = 20 + 9i$

   b. $2(5x + 9) = (10 - 3y)i$

   c. $3(7 - 2x) - 5(4y - 3)i = x - 2(1 + y)i$

5. Express each of the following in $a + bi$ form.
   a. $i^5$
   b. $i^6$
   c. $i^7$
   d. $i^8$
   e. $i^{102}$

6. Express each of the following in $a + bi$ form.
   a. $(1 + i)^2$
   b. $(1 + i)^4$
   c. $(1 + i)^6$

7. Evaluate $f(x) = x^2 - 6x$ for $f(3 - i)$, so when $x = 3 - i$

8. Evaluate $f(x) = 4x^2 - 12x$ for $f(\frac{3}{2} - \frac{i}{2})$, so when $x = \frac{3}{2} - \frac{i}{2}$.

9. Show by substitution that $\frac{5 - i\sqrt{5}}{5}$ is a solution to $5x^2 - 10x + 6 = 0$.

10.

a. Evaluate the four products below.

Evaluate $\sqrt{9} \cdot \sqrt{4}$.

Evaluate $\sqrt{9} \cdot \sqrt{-4}$.

Evaluate $\sqrt{-9} \cdot \sqrt{4}$.

Evaluate $\sqrt{-9} \cdot \sqrt{-4}$.

b. Suppose $a$ and $b$ are positive real numbers. Determine whether the following quantities are equal or not equal.

$\sqrt{a} \cdot \sqrt{b}$ and $\sqrt{-a} \cdot \sqrt{-b}$

$\sqrt{-a} \cdot \sqrt{b}$ and $\sqrt{a} \cdot \sqrt{-b}$