Name:_____

Constructions with Constructions and Straightedge - Part 2

Original Text By Dr. Bradley
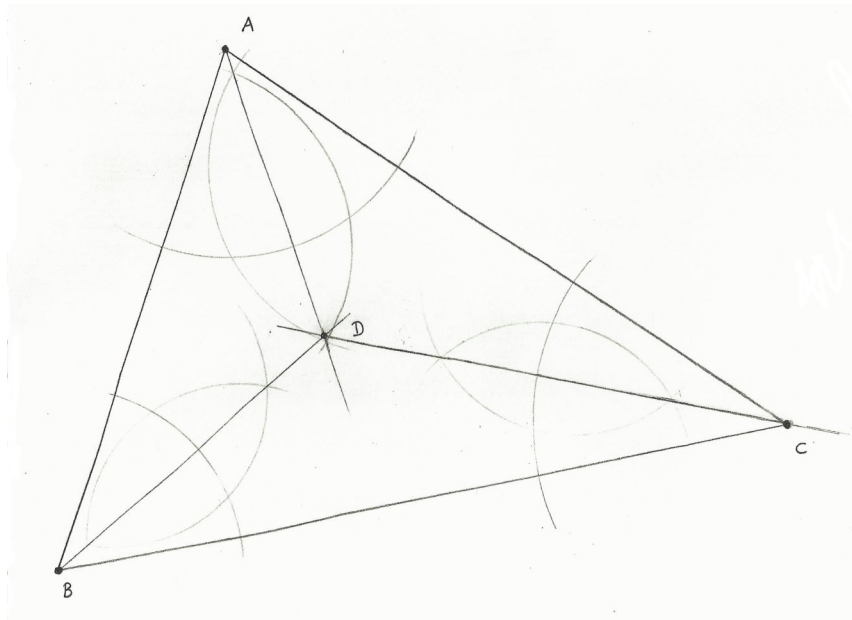Material Supplemented by Mrs.de Nobrega

4.8 Points of Concurrency in a Triangle

Our last four constructions are our most challenging and you will need some practice to construct them with ease and accuracy. They each illustrate a special property of a triangle. First we will perform these constructions with compass and straightedge before investigating them in a dynamic environment with *Geometer's Sketchpad*.

4.8.1 A Triangle and its Angle Bisectors

1. To start, draw three non-collinear points on a plane, and draw the induced lines. The result is a triangle, as explored in Section **??**. Let's call this $\triangle ABC$

2. Construct the bisectors of $\angle A$ and $\angle B$.

3. Construct point D, the point of intersection of the two angle bisectors.

4. Construct the bisector of $\angle C$.

The result should look similar to this drawing:

Explore 4.8.1

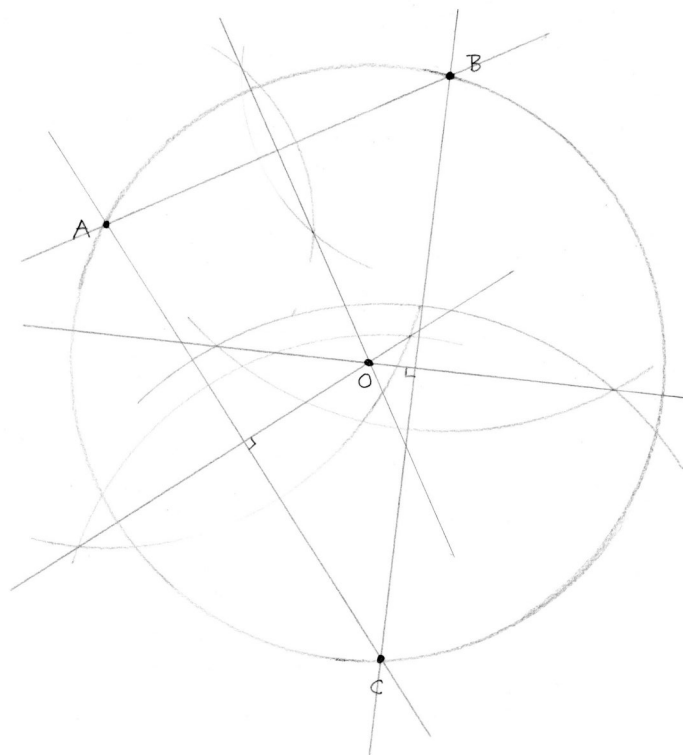1. What do you notice about this third angle bisector?

When three or more lines intersect in a single point we call this point the **point of concurrency**. The point where the three angle bisectors of a triangle intersect is called the **incenter**.

Later in this unit we will investigate the point of intersection of the angle bisectors in a triangle more using *Geometer's Sketchpad.*

4.8.2 A Triangle and its Perpendicular Segment Bisectors.

The next construction is perhaps even more fascinating, as it yields a surprising result.

1. Again, draw three non-collinear points on a plane, and then draw the induced lines. Let's call this $\triangle ABC$

2. Construct the midpoints of the sides and constrcut the three perpendicular bisectors.

3. Construct point O, the point of intersection of the three perpendicular bisectors.

   Again, three lines intersect at one point O, their point of concurrency. But we're not done.

4. Place the pivot of your compass on O and adjust it so that the pencil is on A; then draw the induced circle. If the construction has been done carefully, the circle should intersect B and C as well, as in the figure below.

Explore 4.8.2

1. Three non-collinear points determine a triangle. That is, three such points induce precisely one triangle—no more, no fewer. After practicing this construction a few times and making observations, hypothesize on the relationship between three non-collinear points and circles.
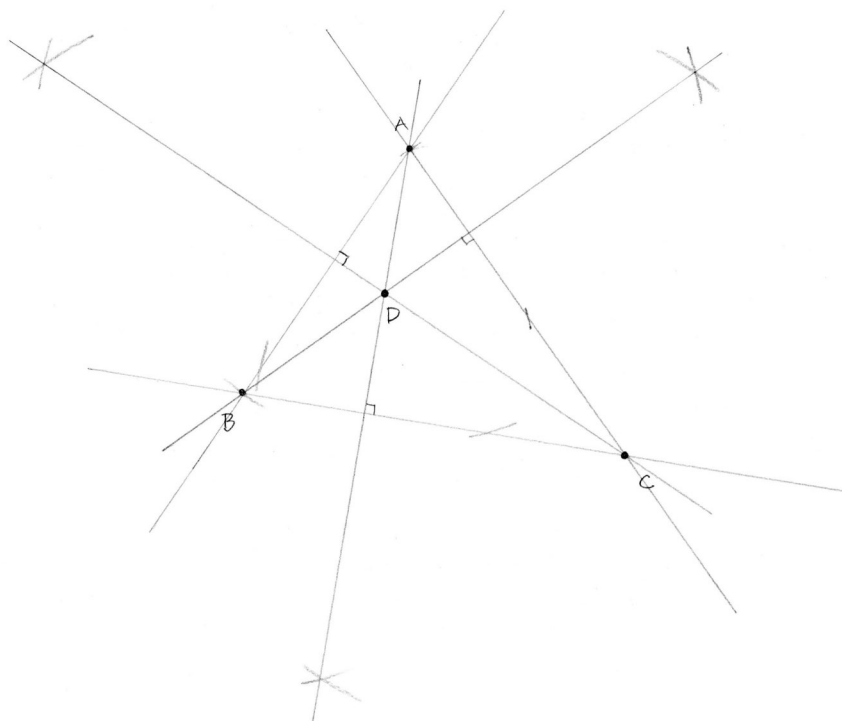
2. Explain how this constrcution can help you find the center of a circle?

3. Explain why the point of concurrency is the center of a circle that intersect all three vertices of the traingle.

The point where the three perpendicular bisectors of a triangle intersect is called the **circumcenter**.

The two previous constructions applied bisection three times. In the next construction, we construct perpendiculars to the sides of a triangle through their opposite vertexes.

4.8.3 A Triangle and its Altitudes

So, again, draw three non-collinear points and the induced lines. To keep things simple I would suggest creating an acute triangle. We will look at other triangles when using *Geometer's Sketchpad*. Now draw the perpendicular to $\overleftrightarrow{AB}$ through C - notice you are constructing a line perpendicular to $\overleftrightarrow{AB}$ through C, a point not on the line. This perpendicular line is called the altitude(length of altitude = height) of the triangle and is not the same line as the perpendicular bisector. Similarly, draw the perpendicular to $\overleftrightarrow{BC}$ through A and the perpendicular to $\overleftrightarrow{AC}$ through B.The construction is shown in the figure below:
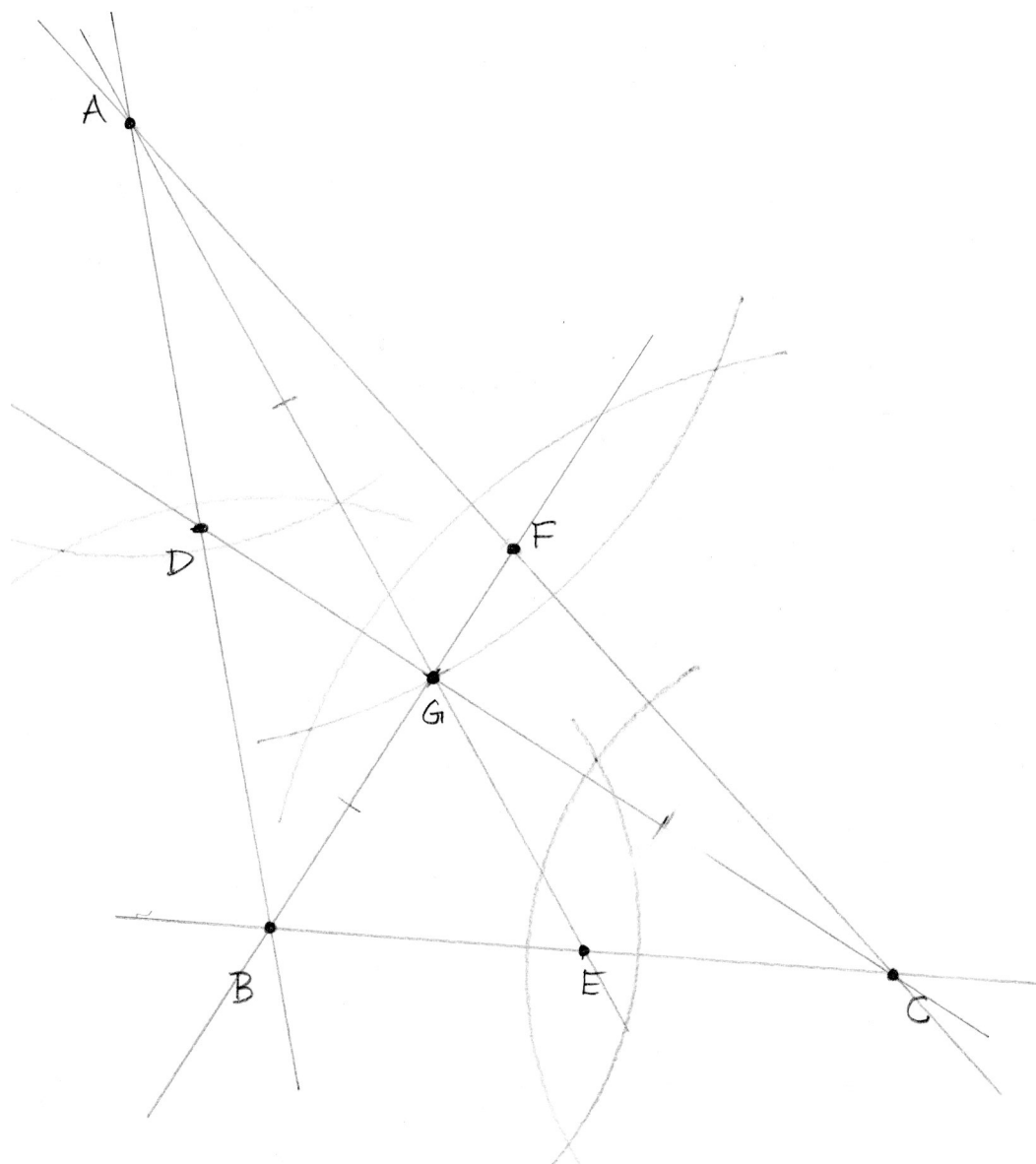

Once again, the three constructed lines are concurrent (at D in the figure). The point where the three altitudes of a triangle intersect is called the **orthocenter**.

We will investigate the intersection of the altitudes of a triangle more using *Geometer's Sketchpad*.

4.8.4   A Triangle and its Medians

One last time, draw three non-collinear points and the induced lines. Then bisect each side of the triangle. Finally, add the line segments between each point of bisection and the opposite vertex of the triangle. The result should look similar to this drawing:

The startling observation—if the construction is carefully done—is that the lines $\overleftrightarrow{AE}$, $\overleftrightarrow{BF}$, and $\overleftrightarrow{CD}$ (called the **medians** of $\triangle ABC$) intersect at one point, G, yet again.

  Explore 4.8.4

  1. Describe the difference between an altitude and a median of a triangle.

2. Use your compass to observe if there is any relationship between the two lengths of the segments that the point of concurrency divides each median into. If you are not sure write "Not sure". You will have another chance to look at the lengths when we use *Geometer's Sketchpad*

The point where the three medians of a triangle intersect is called the **centroid**.

Practice: These four constructions are our most complicated yet. Practice them until you reliably get points of concurrency -the point where three or more lines intersect.

5 So You Want to Be a Geometer?

You have begun your apprenticeship as a geometer. Throughout these lessons, you have developed and practiced several skills of a geometer and, more generally, of a mathematician:

- You have combined basic ideas to construct more complex ones.

- You have seen the distinction between abstract mathematics and concrete representations. The ideas of points, lines, angles, and intersections are abstract; the drawings of them are concrete representations.

- You have learned the difference between an instance and the general case. For example, a particular pair of points and a line is an instance of the general idea, "Two points determine a line."

- You have practiced the key skill of observation.

- You have practiced the key skill of generalizing from instances to a conjecture, a process called "inductive reasoning." We push the frontiers of mathematical knowledge by observing, generalizing, and hypothesizing.

- You have developed explanations for why some conjectures are true, a process called "deductive reasoning." A mathematician explores a hypothesis with such reasoning.

- You have witnessed geometric coincidences that defy your current understanding.

The unexplained - that is what motivates a mathematician. In order to explain the points of concurrency fully, we will use *Geometer's Sketchpad* for exploration in a dynamic geometry environment.